1 **a**
$$\begin{aligned}50^\circ &= \frac{1}{2}x\\ x &= 100^\circ\\ y &= \frac{1}{2}x\\ &= 50^\circ\end{aligned}$$

b
$$y = 360^\circ - 108^\circ = 252^\circ$$
$$x = \frac{1}{2} \times 252 = 126^\circ$$
$$z = \frac{1}{2} \times 108^\circ = 54^\circ$$

c Acute $\angle O = 2 \times 35 = 70^\circ$
$$z = 360^\circ - 70^\circ = 290^\circ$$
$$y = \frac{1}{2} \times 290 = 145^\circ$$

d $O = 180^\circ$
$$x = 360 - 180 = 180^\circ$$
$$y = 90^\circ \text{ (Theorem 3)}$$

e
$$\begin{aligned}3x + x &= 180^\circ\\ 4x &= 180^\circ\\ x &= 45^\circ\\ z &= 2 \times 3x\\ &= 2 \times 3 \times 45^\circ = 270^\circ\\ y &= 360^\circ - 270^\circ\\ &= 90^\circ\end{aligned}$$

2 The opposite angles of a cyclic quadrilateral are supplementary.

a
$$\begin{aligned}x + 112^\circ &= 180^\circ\\ x &= 68^\circ\\ y + 59^\circ &= 180^\circ\\ y &= 121^\circ\end{aligned}$$

b
$$\begin{aligned}x + 68^\circ &= 180^\circ\\ x &= 112^\circ\\ y + 93^\circ &= 180^\circ\\ y &= 87^\circ\end{aligned}$$

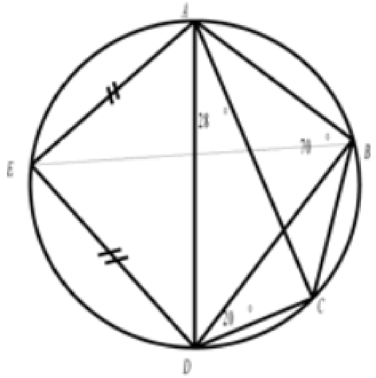
c
$$\begin{aligned}x + 130^\circ &= 180^\circ\\ x &= 50^\circ\\ y + 70^\circ &= 180^\circ\\ y &= 110^\circ\end{aligned}$$

3 Let the equal angles be x°.
$$\begin{aligned}2x + 40^\circ &= 180^\circ\\ 2x &= 140^\circ\\ x &= 70^\circ\end{aligned}$$

The angles in the minor segments will be the opposite angles of cyclic quadrilaterals.

$$180^\circ - 70^\circ = 110^\circ$$
$$180^\circ - 70^\circ = 110^\circ$$
$$180^\circ - 40^\circ = 140^\circ$$

4



In cyclic quadrilateral *ABDE*, $\angle DEA = 110^\circ$

On arc DC, $\angle DBC = 28^\circ$

$\therefore \angle ABC = 70 + 28 = 98^\circ$

Join EB. Equal chords will subtend equal angles at the circumference.

$\therefore \angle ABE = \angle EBD = 35^\circ$

$\angle EAD = 35^\circ$ (also on equal arcs)

On arc BC, $\angle BAC = \angle BDC = 20^\circ$

$\therefore \angle EAB = 35^\circ + 28^\circ + 20^\circ = 83^\circ$

In cyclic quadrilateral $ABDE$,

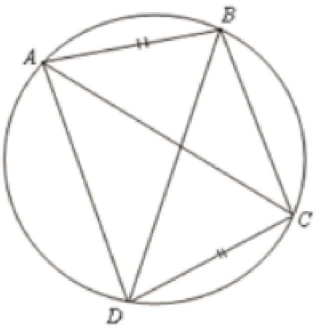
$\angle EDB = 180^\circ - 83^\circ = 97^\circ$

$\therefore \angle EDC = 97^\circ + 20^\circ = 117^\circ$

In cyclic quadrilateral $ABCD$,

$\angle BCD = 180^\circ - (28^\circ + 20^\circ) = 132^\circ$

5



$\angle BAC = \angle BDC$ (subtended by the same arc)

$\angle DAC = \angle BDA$ (subtended by equal arcs)

$\therefore \angle BAC + \angle DAC = \angle BDC + \angle BDA$

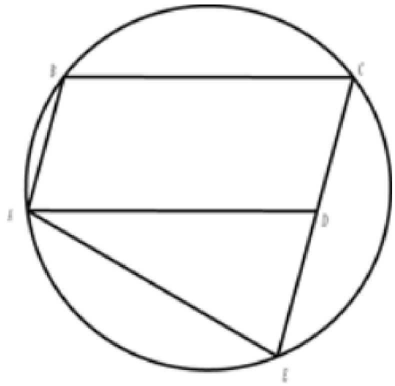
$\angle BAD = \angle ADC$

$\angle ADC + \angle ABC = 180^\circ$ (opposite angles in a cyclic quadrilateral)

$\therefore \angle BAD + \angle ABC = 180^\circ$

BC and AD are thus parallel, as co-interior angles are supplementary

6



$\angle ADE + \angle ADC = 180^\circ$

$\angle ABC = \angle ADC$ (opposite angles in a parallelogram)

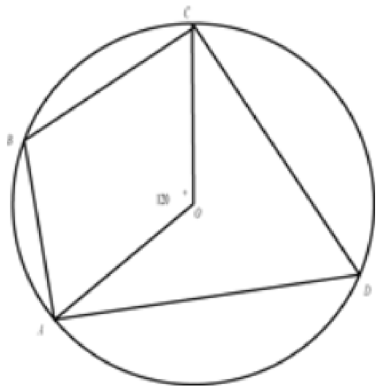
$\therefore \angle ADE + \angle ABC = 180^\circ$

$\angle AED + \angle ABC = 180^\circ$ (opposite angles in a cyclic quadrilateral)

$\therefore \angle ADE = \angle AED$

$AE = AD$

7

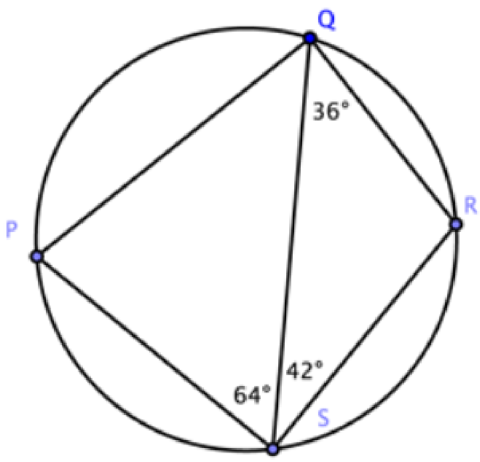


$\angle ADC = \dfrac{120^\circ}{2} = 60^\circ$

If B and D are on opposite sides of AOC, then $\angle ADC = \dfrac{240^\circ}{2} = 120^\circ$.

(Reflex angle $ADC = 360^\circ - 120^\circ$ will be used.)

8



In $\triangle QRS$, $\angle QRS = 102^\circ$ (angle sum of triangle)

$\angle PSR = 64^\circ + 42^\circ = 106^\circ$

$\angle PQR = 74^\circ$ (opposite angles in a cyclic quadrilateral)

$\angle QPS = 78^\circ$ (opposite angles in a cyclic quadrilateral)

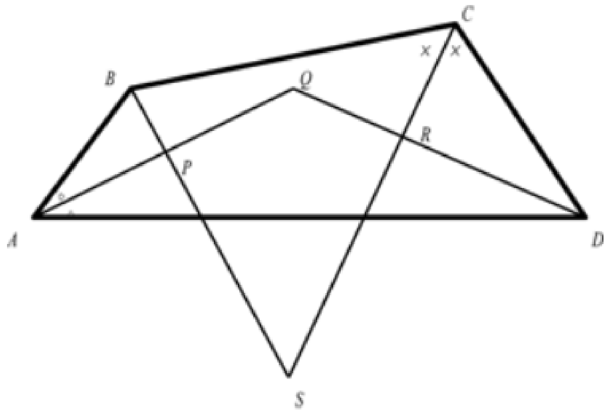
9 The opposite angles in a parallelogram are equal.

In a cyclic parallelogram, the opposite angles will add to 180°.

$\therefore$ the opposite angles equal 90°.

$\therefore$ all angles are 90°, i.e. the parallelogram is a rectangle (subtended by the same arc).

10



In triangle BCS,

$$\angle BSC = 180^\circ - \angle SBC - \angle BCS$$
$$= 180^\circ - \frac{1}{2}\angle ABC - \frac{1}{2}\angle BCD$$

Likewise, in triangle AQD

$$\angle AQD = 180^\circ - \frac{1}{2}\angle BAD - \frac{1}{2}\angle CDA$$

$$\therefore \angle BSC + \angle AQD$$
$$= 180 - \frac{1}{2}\angle ABC$$
$$- \frac{1}{2}\angle BCD + 180^\circ - \frac{1}{2}\angle BAD$$
$$- \frac{1}{2}\angle CDA$$
$$= 360^\circ - \frac{1}{2}(\angle ABC + \angle BCD$$
$$+ \angle BAD + \angle CDA)$$

$$\angle ABC + \angle BCD + \angle BAD + \angle CDA$$
$$= 360^\circ \text{ (angle sum of quadrilateral)}$$

$$\angle BSC + \angle AQD = 360^\circ - 180^\circ$$
$$= 180^\circ$$

$\therefore$ both pairs of opposite angles in $PQRS$ will add to 180°.

$\therefore$ $PQRS$ is a cyclic quadrilateral.